DECAGON DEVICES

Application Note

The KD2 Thermal Properties Analyzer vs. Published Standards

The measurement method and analysis used by the KD2 thermal properties analyzer to obtain thermal conductivity is based on recommendations of several published standards. While the standards differ in their details, the general methods are similar for all of them. The purpose of this note is to give the background for the method, compare the KD2 hardware and measurement procedure with those of the various standards, and provide some justification for the choices made in the KD2 design.

Theory

The method used is generally called the transient line heat source or transient heated needle method. If heat at a constant rate, *q* is applied to an infinitely long and infinitely small "line" source, the temperature response of the source over time can be described by the equation

$$\Delta T = \frac{q}{4\pi k} Ei\left(\frac{-r^2}{4Dt}\right) \qquad (1)$$

where *k* is the thermal conductivity of the medium in which the line is buried, *D* is the thermal diffusivity of the medium, *r* is the distance from the line at which temperature is measured, and *Ei* is the exponential integral. *Ei* is defined in the following equation, and can be approximated by the series shown

$$-Ei(-\alpha) = \int_{\alpha}^{\infty} (1/u)\exp(-u)\,du \qquad (2)$$

$$= -y - \ln\alpha + \alpha - \alpha^2/4 + \ldots$$

in which y = 0.5772... is Euler's constant and $\alpha = r^2/4Dt$.

The terms beyond ln α in the series expansion of *Ei* become negligibly small for long times, and especially when *r* is small and *D* is large, so equation 2 can be approximated as:

$$\Delta T \approx \frac{q}{4\pi k}\left[-y - \ln\left(\frac{r^2}{4Dt}\right)\right] = \frac{q}{4\pi k}\left[\ln t - \ln\left(\frac{r^2}{4DC_E}\right)\right] \qquad (3)$$

where C_E = exp$y$. Thus, after some delay, a graph of ΔT vs. ln$t$ becomes a straight line with slope equal to $q/4\pi k$. Since two points define a straight line, *k* can be computed from

$$k \approx \frac{q(\ln t_2 - \ln t_1)}{4\pi(\Delta T_2 - \Delta T_1)} \qquad (4)$$

This approximation is used in all of the transient line source standard methods for obtaining *k*.

Fifty years ago, when digital computers were unavailable, and computations were done by hand, there may have been some justification for such a simplified method, but there is little justification for continuing to use simplifying assumptions which produce erroneous results if the means are readily available to do better.

Approximating the exponential integral by the logarithm is one assumption made to get to eq. 4, but it isn't the only one. Real probes are neither infinitely long nor infinitely small. In addition, the ambient temperature of the sample is never constant during a measurement; there is always some temperature drift. Fortunately, the solution to the differential equation for finite length and radius probes can be obtained. For a heated cylindrical source of radius a (m) and length $2b$ (m), with temperature measured at its center, the temperature rise during heating is

$$\Delta T = \frac{q}{4\pi \mathrm{k}} \int_{r^2/4Dt}^{\infty} u^{-1} \exp(-u) \exp\left[-(a/r)^2 u\right] I_o(2au/r) erf\left(\frac{b}{r}\sqrt{u}\right) du \tag{5}$$

Here, $I_o(x)$ represents a modified Bessel function of order zero, $erf(x)$ is the error function, and u is an integration variable.

The quantity $\exp\left[-(a/r)^2 u\right] I_o(2au/r)$ approaches unity as a/r approaches 0, and $erf\left(\frac{b}{r}\sqrt{u}\right)$ approaches unity as b/r approaches infinity. In these limits, eq.5 becomes eq. 1.

Error Analysis

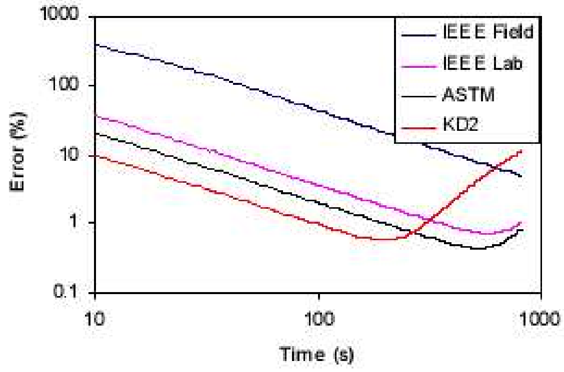
Equation 5 can be used to assess the errors which arise by using eq. 1 or eq. 4 to obtain values for k when finite length and diameter probes are used. The construction of the KD2 probe, as well as those proposed in the standards, is consistent with an assumption that the source radius, a, and the measurement radius, r, are the same. The probe lengths and diameters suggested in the various standards, as well as the dimensions of the KD2 probe are given in Table 1.

Table 1 Needle dimensions suggested in various standards, and Decagon's needle dimensions.

	IEEE Field	IEEE Lab	ASTM	KD2
Length (mm)	2000	100	100	60
Diameter (mm)	8	2.4	1.8	1.27

Figure 1 shows the error in computing k using eq. 4 from data generated with eq. 5. The error shown is the difference between the computed k using eq. 4 and the actual k used in eq. 5 to generate the data, divided by the actual k. The time scale shows the time at which the slope in eq. 4 was computed. Three things are readily apparent from the figure. First, probe size strongly affected error. The larger the probe, the larger the error at a particular time. Second, errors decrease with time, so that even large probes have acceptably small error after sufficiently long time. The third observation is that error starts to increase after sufficient heating time. This is due to the finite probe length. For an infinitely long probe, the error continues to decrease with time. All of the probes in Table 1 are sufficiently long to give negligible error from finite probe length.

Figure 1 Error in the k value computed using eq. 4 as a function of the time at which the slope is computed for the probes shown in Table 1. Properties for the simulation were those of a dry soil. Results are similar for a saturated soil.



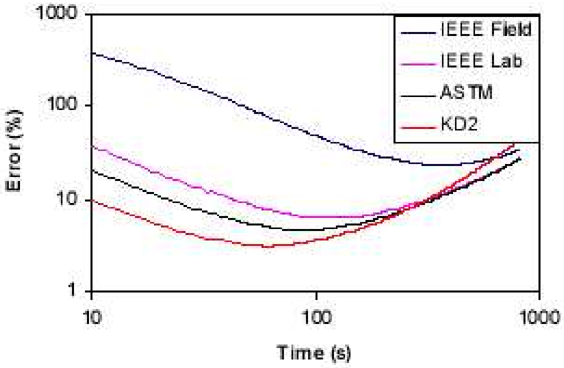
The effect of finite probe diameter on measurement error is always in the direction of overestimating the thermal conductivity. All of the errors shown in Fig. 1 are easily eliminated by calibrating against standards of known thermal conductivity, but probes are often used without calibration. The result is that reported thermal conductivities are often high by 30 to 50%.

All but the field probe reach acceptable error values with 30 to 200 sec. heating. Longer heating times are detrimental in at least two ways. In moist soil, water moves from regions of high temperature to regions of low temperature. The heating of the needle therefore drives moisture from around the needle. This reduces the thermal conductivity in exactly the region where conductivity is being measured. Minimizing heating time reduces the magnitude of this error.

The second effect of long heating times on error is through the effect of temperature drift on the results of the measurement. The method proposed in both the ASTM and IEEE standards is extremely susceptible to temperature drift during the measurement time. Figure 2 shows the effect on error of an extremely small sample temperature drift of –0.0001 C/s. Error is minimized by using short heat times, since the probe heats very little at long times and the effect of drift is relatively larger then.

With that background, we can now compare details of the KD2 measurement and analysis to those of the standards. Table 1 compares needle dimensions of the KD2 to those suggested by the standards. The KD2 needle is shorter and smaller than suggested sizes. Figures 1 and 2 indicate that the smaller needle should give better results if eq. 4 is used for the analysis. In fact, if calibrations are done against standards of known conductivity, all of the needle sizes, except possibly the largest, will give accurate results.

Figure 2 Error in computing *k* as a function of time of slope measurement the probes suggested by several standards, and for the KD2 probe when there is a temperature drift in the material under test of –0.0001 C/s.



Comparison of KD2 and Standards

The ASTM and IEEE standards suggest collecting data with pencil and paper over a 1000 s heat time, plotting the data on semi-log graph paper, selecting a segment of the data by eye that appears to fit a straight line, selecting two points on that line to enter into eq. 4, and computing k from eq. 4. The KD2 collects data at 1 s intervals during a 30 s heating time and a 30 s cooling time. The final 20 points during heating and cooling are used in a simultaneous least squares computation which determines k, while removing effects of temperature drift during the measurement.

Temperature is measured by a 16 bit A to D converter. All of the computations are done by an internal 16 bit microcontroller, and the result is displayed. Because all the computations are done internally, there is no need to record individual temperature values. In addition, forty data points are used to determine the value of k rather than just 2, linear temperature drift effects are removed, and subjectivity inherent in manual or "eye" fitting of data is eliminated. The accuracy of the measurement is verified using known thermal conductivity standards such as glycerol and agar-stabilized water.

Decagon's claim that the KD2 conforms to ASTM D5334 and IEEE 442-1981 standards is based on the fact that the KD2 uses a transient line heat source or transient heated needle method and an approximation to the solution to the differential equation for an infinite line heat source as the method for finding k. The fact that a smaller needle and a shorter heating time are used, and that the analysis is done within a microcontroller, are expected results of improved understanding of the physics, and improving technology. They only improve on the basic specified method.

References

1. ASTM D 5334-00 (2000) Standard test methods for determination of thermal conductivity of soil and soft rock by thermal needle probe procedure. Vol. 04.08, ASTM, 100 Barr-Harbor Dr., West Conshocken, PA 19428-2059.
2. ASTM D 5930-01 (2001) Standard test method for thermal conductivity of plastics by means of a transient line-source technique. Vol 14.02, ASTM, 100 Barr-Harbor Dr., West Conshocken, PA 19428-2059.
3. IEEE STD 442-1981 IEEE guide for Thermal Resistivity Measurements . The Institute of Electrical and Electronics Engineers, Inc. 345 East 47 Street, New York, NY 10017
4. Bristow, K. 2002 . Thermal Conductivity . p1209-1226. Methods of soil analysis. Part 4 Soil Science Society of America, Inc. 677 South Segoe Rd. Madison, WI 53711-1086.

Decagon Devices, Inc.
2365 NE Hopkins Court
Pullman, WA 99163 USA
support@decagon.com
kd2.decagon.com